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About the Poor Decay of Certain Cross-Correlation Functions in the Statistical Mechanics of Phase Transitions in the Static and Dynamical Regime

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We introduce a method to prove poor decay of certain cross-correlation functions which are closely related to the phase transition. The methods apply both to equal and nonequal times, which gives access to the dynamical regime. We establish a criterion which displays openly what happens when the Goldstone picture breaks down. Since no rudiments of translation invariance are needed the treatment covers phases in coexistence like, e.g., liquid–gas interfaces and completely inhomogeneous systems. Furthermore a perhaps surprising connection with the breaking of time reflection invariance of the equilibrium state is established.

KEY WORDS: Phase transitions; cluster properties; phase coexistence; time reflection invariance.

1. INTRODUCTION

In connection with phase transitions which consist of a spontaneous symmetry breaking of a continuous symmetry of the system under discussion, the "Bogoliubov inequality" has been frequently employed to exhibit the showing up of long-range correlations which are present in the pure phases of the system. They have their origin in long-lived collective excitations, the so-called Goldstone modes, which usually come into existence as a coherent effect when the phase transition sets in, producing, e.g., the famous $|k|^{-2}$ singularities in certain systems. See, e.g., Refs. 1–4, 8, or more recently 5 and 15.

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But the Bogoliubov inequality provides access only to a special class of correlations, namely, the so-called autocorrelation functions of the symmetry breaking observable in the static regime, i.e., for equal times. Furthermore, a rudimentary form of translation invariance is needed. Dynamical effects of the symmetry breaking have to be studied with the help of other methods (compare, e.g., Refs. 6 and 7 for quantum statistical mechanics, 9 for classical statistical mechanics). They are much more involved and depend on the special shape of the Goldstone excitation branches under discussion.

Furthermore there are other classes of physically relevant correlation functions like, e.g., the cross correlation between the generator density of the symmetry and the symmetry-breaking observable for equal, respectively, nonequal times, the poor clustering of which is a little bit surprising on various grounds (which will become apparent in the following). In particular we prove rigorously that even in the case where the static clustering is good there is an open set of time values such that the clustering becomes poor for the time difference t_1-t_2 lying in this set. Thus the dynamical influence of the Goldstone mode is rigorously displayed.

In addition to establishing access to a new class of correlation functions we do not need even a rudimentary form of translation invariance. That is, the whole field of phase transitions in inhomogeneous media is also covered. This extends results derived previously in the case of breaking of translation invariance in classical statistical mechanics (cf. Refs. 10 and 11). To mention a few examples, phase transition taking place in an inhomogeneous exterior field (e.g., the numerous superconducting phenomena), the interesting branch of phenomena related to phases in coexistence (i.e., phase boundaries, e.g., liquid–gas interfaces), the various vortex structures of superconductors of the second kind, etc.

As a further surprise time reflection invariance enters the stage within this context, more precisely, the spontaneous breaking, respectively, nonbreaking, of the equilibrium state under time reflection appears to be of relevance. This point will be discussed at the end of the paper.

A short remark concerning time evolution should be appropriate. In this paper we definitely assume the existence of a time evolution α_t acting upon the observables of the system (Heisenberg picture) in order to derive nonstatic results, that is, with A being an observable we have $A(t) = \alpha_t(A)$, α_t being an automorphism. It will depend on the system under discussion whether it is appropriate to consider α_t as being given independently of the concrete state $\langle \cdot \rangle$ of the system or whether α_t is dependent on the state under discussion. In any case, under mild technical assumptions one can implement α_t by a group of unitary operators U_t provided the state is invariant under α_t, i.e.,

$$\langle U_t \cdot A \cdot U_{-t} \rangle = \langle \alpha_t(A) \rangle = \langle A \rangle \qquad \text{for every observable } A \qquad (1)$$

The construction of U_t runs under the name Gelfand–Naimark–Segal construction (see, e.g., Chapter 4 in Ref. 16). Via Stone's theorem the existence of a suitable infinitesimal generator H is guaranteed, the concrete relation of which to the naively given Hamiltonian being, however, another story (compare, e.g., Ref. 13).

2. THE DYNAMICAL NOTION OF SPONTANEOUS SYMMETRY BREAKING AND THE PROPERTY (G)

Without intending to go into all the details, it seems useful to make some remarks about what we consider to be the "Goldstone phenomenon" and certain subtleties of the various assumptions made in this context.

A symmetry is usually a mapping of the algebra of observables onto itself which does commute in a formal sense with the Hamilton operator, that is, it should also leave invariant the related Gibbs state. This is true in the finite volume situation. In the limit $V \to \infty$ one has to make precise what is actually meant by the notion "symmetry." A conserved symmetry is roughly a norm-preserving one-to-one mapping of the algebra of observables which leaves the equilibrium state invariant, that is, with σ denoting the symmetry: (i) $\langle \sigma(A) \rangle = \langle A \rangle$ for all A. But frequently one expects more on physical grounds. The formal invariance of the Hamiltonian is reflected by the invariance of the whole dynamics of the system under the symmetry, that is, (ii) $\langle \sigma(A(t)) \rangle = \langle (\sigma(A))(t) \rangle = \langle \sigma(A) \rangle = \langle A \rangle$. In other words, we have the following important additional property:

Property (G): $\sigma \cdot \alpha_t = \alpha_t \cdot \sigma$.

In the following we will exclusively deal with a continuous one-parameter group of symmetries, σ_s, which can always be accomplished by selecting a suitable subgroup. Then (i) and (ii) together establish what we consider to be a conserved symmetry. The symmetry is dubbed spontaneously broken when (i) no longer holds with property (G) still being fulfilled. That is, invariance of the dynamics, noninvariance of the state. The breakdown of property (i) implies the existence of an observable A with $\langle \sigma_s(A) \rangle \neq \langle A \rangle$.

We should, however, emphasize that this property (G) is by no means an obvious consequence of the formal invariance of the Hamiltonian under the symmetry. Since it will appear that it is exactly the suitable tool to eliminate all the range questions of the interaction in the context of SSB there must be a considerable amount of deeper physics in it. In particular, its exact meaning has to be clarified and in what situations it may be violated.

As to the first point, if the symmetry is spontaneously broken it *cannot* be implemented by a unitary operator in the representation space. So (G) and its breakdown have to be understood as statements about mappings applied to observables $\sigma(A(t)) = (\sigma(A))(t)$ and $\sigma(A(t)) \neq (\sigma(A))(t)$, respectively. The right sides of these relations have a clear meaning since $\sigma(A)$ is also a localized observable if A and hence $\alpha_t(\sigma(A))$, is well defined. In order that $\sigma(A(t))$ also have such an obvious meaning, $A(t)$ should either still be sufficiently localized, which will be the case for sufficiently well-behaved interactions, or certain limiting processes have to be harmless in a certain sense. In the case of long-range interactions one might conceive that the origin of long-range tails in $A(t)$, a result of a strong delocalizing of $A(t)$ as compared with A, can perhaps result in a $\sigma(A(t))$ being different from $(\sigma(A))(t)$.

The physical reason behind the possible breakdown of (G) may be the pushing up of the Goldstone mode, as is known, e.g., from superconductors (cf., e.g., Ref. 12, which is on the other hand perhaps not completely convincing because limits are several times interchanged, which is usually a delicate point in the presence of SSB). Another reason or perhaps the same, expressed only in a more mathematical form, may be the dependence of the dynamics of the observables on the actual state of the system as it is observed in certain mean-field models (see, e.g., Ref. 17, p. 136). Another interesting model in this context is the one-dimensional jellium with states breaking the translation invariance, while nevertheless with an exponential clustering of correlations (Ref. 18). (As to this last point I want to acknowledge a useful discussion with P. Martin and C. Gruber.)

But in any case, since the breakdown of property (G) is probably also the breakdown of the Goldstone phenomenon proper (namely, the existence of long-lived, low-lying Goldstone excitations), we consider it to be the defining relation of the full Goldstone phenomenon. On the other hand, since property (G) is at the core of this phenomenon the mechanisms of its possible breakdown are very interesting and should be studied separately.

We want to condense now the Goldstone phenomenon in a single formula which will allow us to draw definite conclusions from it. Furthermore the impact of property (G) is clearly exhibited. To this end we use the infinitesimal analog of $\langle\sigma_s(A)\rangle \neq \langle A\rangle$.

So let Q be the formal generator of the one-parameter symmetry group under discussion and A the observable which displays the symmetry breaking. (Note that Q does not exist as a well-defined operator in the representation space when the symmetry is spontaneously broken.) With $q(x,t)$ the generator density we have

$$\lim_{V\to\mathbb{R}^3}\int_V \langle [q(x,t),A]\rangle\, d^n x = c \neq 0 \qquad \text{(and independent of } t!\text{)} \tag{2}$$

The time independence of (2) is the crucial point in this context, which is frequently not clearly stated since most of the papers are dealing exclusively with the $t=0$ case, that is, the static case. The time independence can be easily seen as follows: Time invariance of the equilibrium state and property (G) yield

$$\langle \sigma_s(A(t))\rangle = \langle \alpha_t \cdot \sigma_s(A)\rangle = \langle \sigma_s(A)\rangle \tag{3}$$

SSB implies the existence of an observable A such that $\langle \sigma_s(A)\rangle \neq \langle A\rangle$, hence

$$\langle \sigma_s(A(t))\rangle = \langle \alpha_t \cdot \sigma_s(A)\rangle = \langle \sigma_s(A)\rangle \neq \langle A\rangle \tag{4}$$

and in differentiated form (with A appropriately chosen such that $d/ds|_{s=0}\langle \sigma_s(A)\rangle \neq 0$),

$$\begin{aligned}
&\lim_{V\to\infty}\left\langle \left[\int_V q(x,0)\,dx, A(-t)\right]\right\rangle \\
&\quad = \lim_{V\to\infty}\left\langle \left[\int_V q(x,t)\,dx, A\right]\right\rangle = \frac{d}{ds}\bigg|_{s=0}\langle \sigma_s(A(-t))\rangle \\
&\quad = \frac{d}{ds}\bigg|_{s=0}\langle \alpha_{-t}\cdot \sigma_s(A)\rangle = \lim_{V\to\infty}\left\langle \left[\int_V q(x,0)\,dx, A\right]\right\rangle = c \quad \blacksquare
\end{aligned} \tag{5}$$

That a breakdown of property (G) will be connected with another type of long-range correlation can be seen as follows. Let us assume that the expression (2) is time dependent, and (for simplicity) that the generator density of the symmetry fulfils a local conservation law:

$$\partial_t q(x,t) = -\nabla \cdot j(x,t) \tag{6}$$

Assuming that it is allowed to interchange the limit and differentiation with respect to t we get

$$\begin{aligned}
\partial_t \int \langle [q(x,t),A]\rangle\, d^n x &= -\lim_{R\to\infty}\int_{K_R}\langle [\nabla\cdot j(x,t),A]\rangle\, d^n x \\
&= -\lim_{R\to\infty}\int_{\partial K_R}\langle [j(x,t),A]\rangle\, d^{n-1}\mathbf{o}
\end{aligned} \tag{7}$$

In this way, the possibility of a breakdown of (G) seems to be linked with a certain specific long-range correlation for $t \neq 0$.

3. THE LONG-RANGE x BEHAVIOR OF $\langle q(x,t)A\rangle^T$ IN QUANTUM STATISTICAL MECHANICS

We will start with the simpler quantum mechanical case. The system is allowed to be completely spatially inhomogeneous. With $q(x,t)$ the generator density of the symmetry and A the symmetry-breaking local observable, we want to show that $\langle q(x,t)A\rangle^T$ cannot be an integrable function in x. To this end we will work with the Fourier transform of the above quantity, $J_{qA}(k,\omega)$, which in general in the thermodynamic limit will be a distribution. (In the translation-invariant case it is a measure; see, e.g., Refs. 6 and 9.)

In an equilibrium state we have the KMS property, which can be given the form

$$G_{qA}(k,\omega) = (1 - e^{-\beta\omega})J_{qA}(k,\omega) \tag{8}$$

with G_{qA} the Fourier Transform of $\langle[q(x,t),A]\rangle$. As usual (see, e.g., Ref. 6 and further references therein), the volume integration $\int_V q(x,t)\,dx^n$, $V\to\infty$, is performed by an integration over a set of sufficiently smooth functions $\{f_R(x)\}$:

$$f_R(x) := \begin{cases} 1 & \text{for} \quad |x| \leqslant R \\ 0 & \text{for} \quad |x| \geqslant R+\epsilon \end{cases} \tag{9}$$

Now let us assume that $\langle q(x,t)A\rangle^T$ is ϵL^1 with respect to x, hence the same does hold for $\langle[q(x,t),A]\rangle$. This entails that the Fourier transform with respect to x, $\int e^{i\omega t}J_{qA}(k,\omega)\,d\omega$, is a continuous function in k. The same does hold for $\int(1-e^{-\beta\omega})e^{i\omega t}J_{qA}(k,\omega)\,d\omega$. Performing the limit $V\to\infty$ in (4) with the above class of functions $\{f_R\}$ corresponds to integrating with a class of functions $\{\tilde{f}_R\}$ in k space, the directed system converging toward a δ function $\delta(k)$. Under the above assumption this limit is well defined since the functions under discussion are continuous in k. Thus we have

$$\begin{aligned} c &= \lim_{R\to\infty} \int d^n k\, \tilde{f}_R(k) \int d\omega\, e^{i\omega t}(1-e^{-\beta\omega})J(k,\omega) \\ &= \int d^n k\, \delta(k) \int d\omega e^{i\omega t}(1-e^{-\beta\omega})J(k,\omega) \\ &= \int d\omega\, e^{i\omega t}(1-e^{-\beta\omega})J(0,\omega) \end{aligned} \tag{10}$$

(where we dropped for simplicity the indices q, A). c was shown to be independent of t; hence the Fourier transform with respect to t is simply $c\cdot\delta(\omega)$:

$$(1-e^{-\beta\omega})J(0,\omega) = c\cdot\delta(\omega) \tag{11}$$

Since $(1-e^{-\beta\omega})\neq 0$ for $\omega\neq 0$ this entails that $J(0,\omega)$ is a distribution concentrated in $\omega = 0$, therefore $J(0,\omega)$ has the general structure:

$$J(0,\omega) = \sum_n c_n \delta^{(n)}(\omega) \tag{12}$$

From (11) we can infer

$$J(0,\omega) = c_0\delta(\omega) + c_1\delta^{(1)}(\omega) \tag{13}$$

with $c_1 = -\beta^{-1} \cdot c$. Our next aim is to show that c_1 is zero.

We have

$$\int d^n x \, \langle q(x,t)A\rangle^T = \int d^n x \int e^{i\omega t} \langle q(x,0)\, dE_\omega A\rangle^T$$
$$= \int e^{i\omega t} \int d^n x \, \langle q(x,0) dE_\omega A\rangle^T, \tag{14}$$

where dE_ω is the spectral measure of the time evolution. (We can interchange the dx and dE_ω integration since dE_ω is a finite measure and $\langle q(x,t)A\rangle^T$ is ϵL^1 with respect to x.) Thus we have the identity

$$J(0,\omega)\, d\omega = \int \langle q(x,0)\, dE_\omega A\rangle^T d^n x \tag{15}$$

that is, $J(0,\omega)$ is a measure with respect to ω. With (13) we can conclude $J(0,\omega) = c_0\delta(\omega)$ because $\delta^{(1)}(\omega)$ is evidently not a measure. (11) entails $(1 - e^{-\beta\omega})c_0 \cdot \delta(\omega) = 0 = c \cdot \delta(\omega)$, hence $c = 0$ and $\lim_{V\to\infty}\langle[\int_V q(x,0)\, d^n x, A]\rangle = 0$. In other words, there is no SSB. We have the following result:

Theorem 1. With $q(x,t)$ the generator density of the spontaneously broken continuous symmetry, A the symmetry-breaking observable, $\langle q(x, t)A\rangle^T$ is not integrable with respect to x for a set of t's with Lebesgue measure $\neq 0$.

4. $\langle q(x,t)A\rangle^T$ IN CLASSICAL STATISTICAL MECHANICS

The situation in classical statistical mechanics is slightly more complicated since Poisson brackets are not so easy to handle as commutators. As a general frame of conceptual reference we refer to Ref. 9 and further references there. Continuous symmetries are assumed to act via exponentiation of Poisson brackets with suitable generators. Correspondingly we will discuss the expressions

$$\langle\{q(x,t), A\}\rangle, \langle q(x,t)A\rangle^T \qquad \text{with}$$

$$\{q(x,t), A\}(X) := \sum_{j=1}^{\infty} \left(\frac{\partial q}{\partial q_j} \frac{\partial A}{\partial p_j} - \frac{\partial q}{\partial p_j} \frac{\partial A}{\partial q_j} \right) \tag{16}$$

X being a particle configuration in phase space.

Our starting point will be the so-called dynamical KMS property of classical statistical mechanics which reads in Fourier transform form

$$G_{qA}(k,\omega) = -\beta\omega J_{qA}(k,\omega) \tag{17}$$

with $G(k,\omega)$ the Fourier transform of $\langle\{q(x,t),A\}\rangle$, $J(k,\omega)$ the Fourier transform of $\langle q(x,t)A\rangle^T$, [see, e.g., formula (7) of Ref. 9]. A minor difference as compared with Chapter 3 arises from the fact that $\langle\{q(x,t), A\}\rangle$ is not simply $\langle q(x,t)A\rangle^T - \langle Aq(x,t)\rangle^T$. In Chapter 3 it was sufficient to assume integrability of $\langle q(x,t)A\rangle^T$ to arrive at a contradiction. Here some additional remarks are necessary.

In order that we can perform the limit $R\to\infty$ in

$$c = \lim_R \int \tilde{f}_R(k)\left[\int e^{i\omega t}G(k,\omega)\,d\omega\right]dk \tag{18}$$

under the integral $\int e^{i\omega t}G(k,\omega)d\omega$ has to be continuous in $k=0$, in other words, it is sufficient to assume $\langle\{q(x,t),A\}\rangle$ to be ϵL^1 with respect to x. As already indicated, this does not follow from $\langle q(x,t)A\rangle^T\epsilon L^1$ in the realm of classical statistical mechanics. On the other hand, since $\{f_R(x)\}$ simulate the volume integration $V\to\infty$ the limiting value c should not depend on the detailed form of $f_R(x)$. It should be sufficient that $\{f_R(x)\}$ approximate the function $:=1$ in a weak sense in the limit $R\to\infty$; hence $\tilde{f}_R(k)\to\delta(k)$. In particular, the functions $f_R(x)$ need not be rotationally symmetric. Hence this independence makes it highly implausible that the finite value c comes about by an artificial destructive oscillating behavior of the various contributions of the integral while the function itself is not summable. So we should be allowed to take it for granted that $\langle\{q(x,t),A\}\rangle$ is integrable in x. The long-range character should again come into play in the two-point function $\langle q(x,t)A\rangle^T$.

Proceeding now in the same way as in Chapter 3 we arrive at

$$c\cdot\delta(\omega) = G(0,\omega) = -\beta\omega\cdot J(0,\omega) \tag{19}$$

With $\langle q(x,t)A\rangle^T$ integrable we have $J(0,\omega) = c'\cdot\delta(\omega)$ and therefore $c=0$, thus again no symmetry breaking.

Theorem 2. With $q(x,t)$, A having the meaning of Theorem 1, and provided that we can perform the limit in (18) under the integral, $\langle q(x,t) A\rangle^T$ is not integrable with respect to x for at least an open interval of t's when the symmetry is spontaneously broken.

5. ANOTHER TYPE OF DECAY OF CORRELATION WITH A UNIVERSAL CLUSTERING BEHAVIOR

In this chapter we will show that in addition to the discussion above systems with SSB display another kind of long-range correlation of a fairly universal character, that is, there is always a type of decay of correlation only like $|x|^{-(n-1)}$, dimension $= n$, irrespective of the physical model under discussion. We will perform the calculation for the quantum statistical case. As to classical statistical mechanics, this phenomenon is discussed in Ref. 9, Th. 3, and Ref. 10.

So let $q(x,t)$ be the local generator density, $Q_R := \int_{x<R} q(x,0)\, d^n x$. We assume the interaction to be of finite range. Let H_R be the Hamiltonian restricted to $\{x, |x| < R\}$. This means especially that in the interaction part interaction only between points x_i, $|x_i| < R$ is to be taken into account. Let A again be the symmetry-breaking observable, localized in the finite region Λ. With $q(x)$ local, $\langle [q(x,0), A] \rangle$ has x support contained in Λ. For equilibrium states the relation

$$\langle [q(x), A] \rangle = \left\langle \int_0^\beta ds\, A_{is} [q(x), H] \right\rangle \tag{20}$$

holds, with $A_{is} := e^{-sH} A e^{sH}$. Hence the right-hand side is $\neq 0$ only for $x \in \Lambda$.

Remark. We want to mention that as long as A is not an analytic element with respect to H it is not obvious that A_{is} is well defined for all $0 < s < \beta$ since e^{sH} is unbounded. It can be shown, however, that it is well defined for $s < (1/2)\beta$ in a concrete representation as above (see, e.g., Ref. 13, Chapter 5.4.). Furthermore, while A is localized in a finite region Λ, this should usually not be the case for the analytic continuation A_{is}, since the suppression of high frequencies by the $e^{-s\omega}$ factor in Fourier space has a delocalizing effect in coordinate space. As to this point, the situation is simpler in classical statistical mechanics where the analogous quantity is $\langle A \{q(x), H\} \rangle$.

We want to show now that, while the expression in (20) is localized in Λ, exhibiting thus the short-range order aspect of SSB, it nevertheless contains also the long-range order of SSB which is hidden in (20) by means of a peculiar mutually cancellation of terms having a long-range correlation. To this end we write (with d the range of the interaction)

$$[Q_R, H] = [Q_R, H_{R+d}] = [Q_{R+d}, H_{R+d}] - [Q_{R+d} - Q_R, H_{R+d}] \tag{21}$$

The first bracket on the right-hand side equals $[Q, H_{R+d}]$ with Q taken in a formal sense, acting upon observables via the commutator. It is the typical feature of SSB that $[Q, H_R] = 0$, while even in the limit $\lim_{R\to\infty}$ $[Q_R, H] \neq 0$. This is the relic of the formal but incorrect statement $[Q, H] = 0$ (as a concrete example take, e.g., the Heisenberg ferromagnet where these relations can explicitly be verified).

With (21) we can now conclude

$$c = \lim_R \langle [\, Q_R, A\,] \rangle$$

$$= \lim_R -\int \chi_{R+d,R}(x) \left\langle \int_0^\beta A_{is}\, ds \cdot \left[q(x), H_{R+d} \right] \right\rangle dx \tag{22}$$

with $\chi_{R+d,R}$ the characteristic function of the set $\{x, R < |x| < R+d\}$. Hence, while $\langle [\, Q_R, A] \rangle = \int \chi_R \langle [q(x), A] \rangle\, dx = \int \chi_R \langle \int_0^\beta A_{is}\, ds\, [q(x), H] \rangle\, dx$ is an integral over a function having its support in the fixed domain Λ, it is equal to the integral over a function with support in $\{R < |x| < R+d\}$ with $R \to \infty$! This comes about by a mutual cancelling of terms in $\langle \int_0^\beta A_{is}\, ds\, [q(x), H] \rangle$ having long-range correlation. This balancing is destroyed in $\langle {}_0^\beta A_{is}\, ds\, [q(x), H_{R+d}] \rangle$.

Owing to the local character of $q(x)$ and the finite range of the interaction $B_R(x) := [q(x), H_{R+d}]$ has its support in a ball $K_d(x)$ around x with radius d. Thus we have

$$c = \lim_R \int_{R<|x|<R+d} \left\langle \int_0^\beta A_{is}\, ds \cdot B_R(x) \right\rangle dx \tag{23}$$

Assuming now that $|\langle \int_0^\beta A_{is}\, ds\, B_R(x) \rangle| < c' \cdot |x|^{-(n-1+\epsilon)}$ we would get $c = 0$, that is, we can infer the following.

Theorem 3. Assuming SSB of a continuous symmetry with $q(x,t)$ the generator density, A a symmetry-breaking observable, $B_R(x)$ defined in (23), we have a long-range correlation between $\int_0^\beta A_{is}\, ds$ and $B_R(x)$, the decay of which is weaker than $|x|^{-(n-1+\epsilon)}$ for all $\epsilon > 0$ (a finite range of the interaction assumed).

Remarks. (i) Since we have in (23) an equality of both sides we are better off than in the case of the Bogoliubov inequality. One should expect in fact a behavior $\sim |x|^{n-1}$ instead of a $\leqslant$. (ii) The R dependence of the quantity $B_R(x)$ is quite trivial. It has its origin simply in the fact that the intersection of $K_d(x)$ and $\{x \,|\, \chi_{R+d,R} \neq 0\}$ depends on the position of x in the interior of $\{x \,|\, \chi_{R+d,R} \neq 0\}$. For example, for lattice systems one can easily extract a quantity from $B_R(x)$, which is then a finite sum, which does not have this R dependence and which still displays the long-range order.

6. SOME SKETCHY REMARKS ABOUT THE IMPACT OF TIME REFLECTION SYMMETRY

It seems to be a natural and harmless question wether the point $t = 0$ always belongs to this set for which $\langle q(x,t) \cdot A \rangle$ displays poor clustering. But this is physically quite subtle. It is related to the breaking of an additional symmetry T, namely, time inversion invariance. When T is conserved we can show that $\langle q(x,0)A\rangle^T = 1/2\ \langle[q(x,0),A]\rangle$ holds, which yields an x support contained in support A. If T is also spontaneously broken this relation will usually no longer hold.

Both situations will actually occur. For example, for a crystal T is conserved, hence $\langle q(x,0)\cdot n(0)\rangle^T$ is well behaved, where q is the generator density of translations, n the particle density. For a classical crystal the situation is even more striking. We proved in Ref. 10, Chapter 3) that $\langle q(y,0)\cdot n(x,0)\rangle^T \equiv 0$, while we know that there exists at least an open set of t's such that $\langle q(y,t)\cdot n(x,0)\rangle^T$ is not integrable!

On the other hand there are many systems with T being broken, e.g., all the magnetic systems below the critical temperature, Bose liquids in the superfluid phase, etc. For example, for the free Bose gas can rigorously prove that $\langle q(x,0)\cdot A\rangle^T$ goes to zero like $|x|^{-1}$. That is, for T conserved we have a dynamical effect of the Goldstone mode since clustering is poor only for $t \neq 0$, whereas with T broken we expect also a weak static decay of correlations. These interesting phenomena shall, however, be discussed in more detail elsewhere.

The results of Chapter 5 are connected with the influence of boundary terms on bulk properties of the medium in the physical region where several phases can coexist. In this context we want to mention also Ref. 14.

REFERENCES

1. N. N. Bogoliubov, *Phys. Abh. S. U.* **6**:1, 113, 229 (1962).
2. H. Wagner, *Z. Phys.* **195**:273 (1966).
3. N. D. Mermin and H. Wagner, *Phys. Rev. Lett.* **17**:1133 (1966).
4. P. C. Hohenberg, *Phys. Rev.* **158**:383 (1966).
5. P. A. Martin, preprint, 1981, EPF Lausanne (to appear in *Nuov. Cim.*).
6. M. Requardt, *J. Phys. A* **13**:1769 (1980).
7. M. Requardt, unpublished material.
8. N. D. Mermin, *J. Math. Phys.* **8**:1061 (1967).
9. M. Requardt, *Z. Phys. B* **36**:187 (1979).
10. M. Requardt, preprint, 1981, Göttingen.
11. C. Gruber, P. A. Martin, and C. Oguey, *Commun. Math. Phys.* **84**:55 (1982).
12. R. V. Lange, *Phys. Rev. Lett.* **14**:3 (1965).
13. O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics II* (Springer, New York, 1981).
14. H. Narnhofer, preprint, Wien, August 1981.
15. L. Landau, J. F. Perez, and W. F. Wreszinski, (to appear in *J. Stat. Phys.* 1982).
16. D. Kastler, D. W. Robinson, and J. A. Swieca, *Commun. Math. Phys.* **2**:108 (1966).
17. W. Thirring, *Quantenmech. grosser Syst.*, Springer, New York (1980).
18. H. Kunz, *Ann. Phys.* N.Y. **85**:305 (1974).